Set Theory

ξ is the 'universal' set (i.e. the largest set in the context of the question). This can be infinite (e.g. "The multiples of 3") or finite (e.g. "The integers between zero and five")
A, B, C etc. denote sets (i.e. a collection of elements).
$n(A)$ means the number of elements in set A.
$3 \in A$ means 3 is a member of the set A.
$4 \notin B$ means 4 is not a member of set B.
$A \subset B$ means A is a subset of set B (i.e. every element of A lies within B).
$\emptyset$ denotes the empty set (i.e. the set with nothing in it). It is also denoted $\{\ \}$.
$A \cap B$ means the intersection of A and B (i.e. the elements that lie in both A and B).
$A \cup B$ means the union of A and B (i.e. the elements that lie in either A or B).
A' means not A (any element in ξ not in A lies in A'). $A \cup A' \equiv \xi$.

Patrons are reminded that most problems are most easily attempted with a Venn diagram. Patrons are also reminded that if they use a letter (x, say) then they must define the letter. For example "Let x be the number of students who passed both maths and physics"

1. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 6, 7\}$ and $B = \{2, 3, 4, 7, 8\}$.
   (a) A'. $\boxed{\{2, 4, 8, 9\}}$
   (b) $A \cup B$. $\boxed{\{1, 2, 3, 4, 5, 6, 7, 8\}}$
   (c) $A \cup B'$. $\boxed{\{1, 3, 5, 6, 7, 9\}}$
   (d) $n(B)$. $\boxed{5}$
   (e) $n(A' \cup B')$. $\boxed{7}$
   (f) Is $8 \in A'$? $\boxed{\text{Yes}}$
   (g) Is $11 \in A'$? $\boxed{\text{No}}$
2. $\xi = \{\text{the letters of the alphabet}\}$, $A = \{\text{vowels}\}$ and $B = \{b, c, e, r, t, z\}$.
   (a) $A \cup B$. $\boxed{\{a, b, c, e, i, o, r, t, u, z\}}$
   (b) $A \cap B$. $\boxed{\{e\}}$
   (c) $n(A')$. $\boxed{21}$
   (d) $n((A \cup B)')$. $\boxed{16}$
3. $\xi = \{\text{positive integers}\}$, $A = \{\text{prime numbers}\}$ and $B = \{\text{odd numbers}\}$.
   (a) B'. $\boxed{\{2, 4, 6, 8, \dots\}}$
   (b) $n(A \cap B')$. $\boxed{1}$
4. All of 40 students either take the bus or ride a bike to school. If 26 only take the bus and 10 only ride a bike, how many take both the bus and ride a bike? $\boxed{4}$
5. In Albuquerque, New Mexico, all the residents can speak either English or Spanish. 73 percent of people speak English and 54 percent speak Spanish. What percentage of of the inhabitants speak both languages? $\boxed{27\%}$
6. All 250 students in a school learn either Latin or Greek. If 200 study Latin and 180 study Greek, how many study just Greek? $\boxed{50}$
7. In a class of 24 boys, 16 have an iPhone and 13 have a Playstation. 5 boys have neither. How many boys have both an iPhone and a Playstation? $\boxed{10}$

8. In a group of 70 billionaires, each owns either a Ferrari or an Aston Martin (or both). 40 own an Aston Martin and 12 own both. How many own a Ferrari? $\boxed{42}$

9. A group of 54 people are surveyed as to whether they watch 'Breaking Bad' on TV and whether they watch 'Dexter'. 30 say they watch Breaking Bad. 20 say they watch Dexter. 10 say they watch neither. How many people watch Breaking Bad, but not Dexter? $\boxed{24}$

10. 65 students took exams in both maths and English. 41 passed maths and 37 passed English. Five students passed neither exam. How many passed both subjects? $\boxed{18}$

11. In a school year group of 75, 55 play rugby and 39 play football. If 7 students play neither, how many students play just rugby? $\boxed{29}$

12. In a certain examination, all candidates took at least one exam in Maths, English and French. 72 offered Maths, 64 English and 62 French. 18 offered both Maths and English, 24 Maths and French and 20 English and French. 8 candidates offered all three subjects. How many candidates were there for the examination? $\boxed{144}$

13. In a survey at an airport, all travellers had travelled to France, Spain or Germany. 55 said that they had been to Spain, 53 to France and 79 to Germany. 18 had been to Spain and France, 17 to Spain and Germany, and 25 to France and Germany, while 10 had been to all three countries. How many travellers had been to only one country? $\boxed{97}$

14. In a school, every student had applied to at least one of Cambridge, Oxford or London University. 118 had applied to Cambridge, 98 to Oxford and 94 to London. 42 had applied to both Oxford and Cambridge, 24 to London and Cambridge and 34 for London and Oxford. 8 had applied to all three. How many pupils are at the school? $\boxed{218}$

15. Of 40 people on holiday to the East, all visited Hong Kong, 30 visited Beijing and 19 visited Bangkok. How many visited all three cities? $\boxed{9}$

16. Of 125 adults (all of who listen to at least one of Jazz, Opera or Gangsta Rap), 55 listen to Jazz, 50 listen to Opera and 60 listen to Gangsta Rap. 15 listen to both Opera and Gangsta Rap, 20 listen to Jazz and Opera and 10 listen to Jazz and Gangsta Rap. How many listen to all three types of music? $\boxed{5}$

17. In a form of 24 girls, 16 have a puppy and 9 straighten their hair. If x girls have a puppy and straighten their hair and y have neither a puppy nor straighten their hair, what are the smallest and largest possible values of x and y? $\boxed{x_{\min} = 1, y_{\min} = 0, x_{\max} = 9, y_{\max} = 8}$